

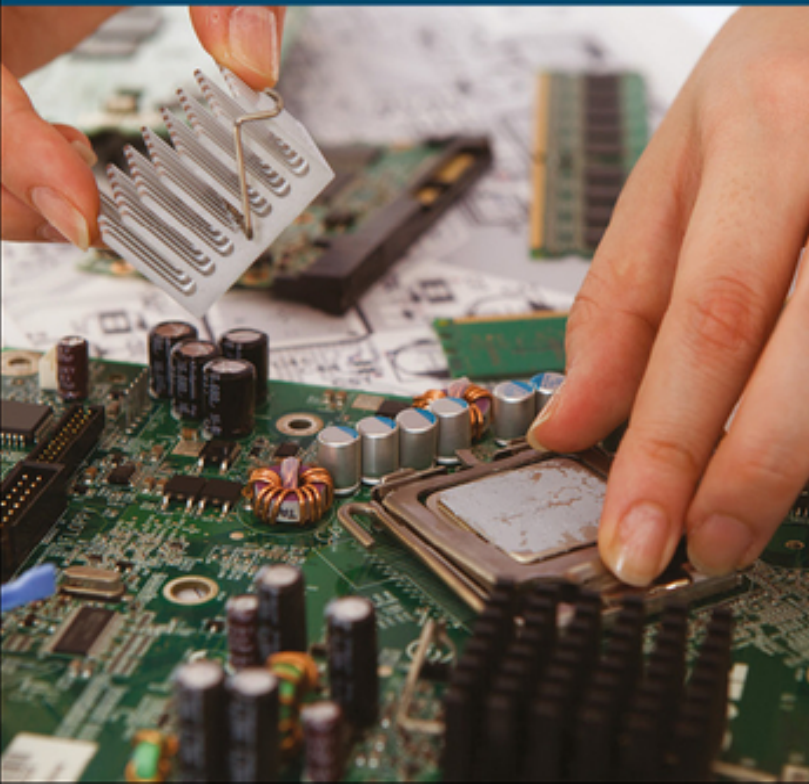


CALTER • CALTER • WRAIGHT • WHITE

THIRD CANADIAN EDITION

# TECHNICAL MATHEMATICS

*with* CALCULUS

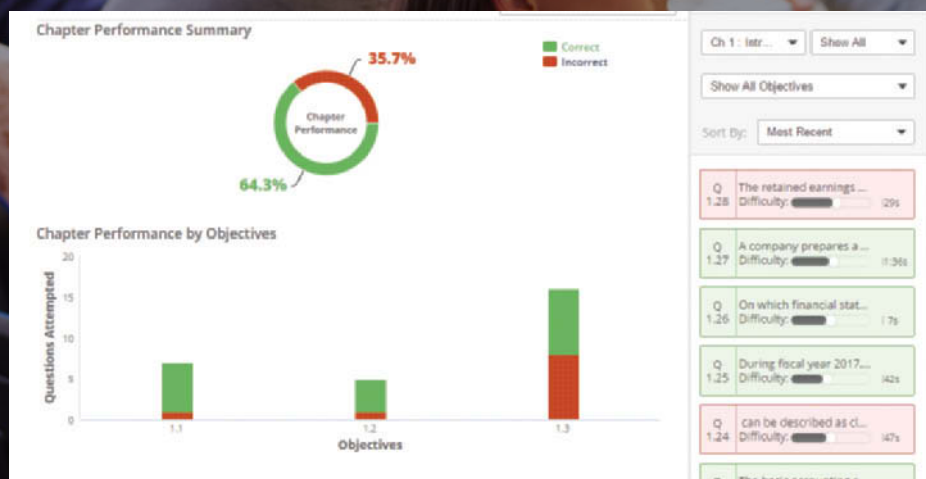






## A personalized, adaptive learning experience.

WileyPLUS with ORION delivers easy-to-use analytics that help educators and students see strengths and weaknesses to give learners the best chance of succeeding in the course.



### Identify which students are struggling early in the semester.

Educators assess the real-time engagement and performance of each student to inform teaching decisions. Students always know what they need to work on.



### Help students organize their learning and get the practice they need.

With ORION's adaptive practice, students quickly understand what they know and don't know. They can then decide to study or practice based on their proficiency.



### Measure outcomes to promote continuous improvement.

With visual reports, it's easy for both students and educators to gauge problem areas and act on what's most important.

## Streamlined Learning Objectives

Easy to follow learning objectives help students make the best use of their time outside of class. Each learning objective is addressed by reading content, watching educational videos, and answering a variety of practice questions, so that no matter where students begin their work, the relevant resources and practice are readily accessible.

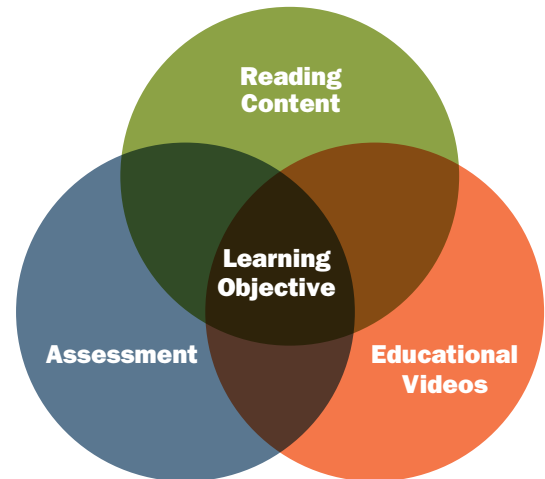
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## Functions

\*\*\* OBJECTIVES \*\*\*

When you have completed this chapter, you should be able to:

- Distinguish between relations and functions and define the range and domain of a function.
- Identify implicit functions, explicit functions, dependent variables, and independent variables.
- Identify and solve composite and inverse functions.



## Review and Practice

Developing effective problem-solving skills requires practice, relevant feedback, and insightful examples with more opportunities for self-guided practice.

The screenshot shows a video player interface. On the left, there are navigation links for 'CHAPTER RESOURCES', 'SOLUTION WALKTHROUGH', 'PRACTICE', and 'APPENDIX'. The main area displays a grid with handwritten mathematical work. It includes the slope formula  $m = \frac{y_2 - y_1}{x_2 - x_1}$  and the point-slope formula  $y - y_1 = m(x - x_1)$ . The final result is  $y = -2x - 8 + 9$ . A small video thumbnail of a person is visible in the top right corner of the grid.

### Review and practice opportunities in WileyPLUS include:

- Solution Walkthrough Videos
- Math Skills Review
- Audio Glossary
- Flashcards
- Crossword Puzzles
- Powerpoint lecture slides
- Practice Quizzes
- Test Bank
- Instructors' Manual

Narrated by the authors of the Canadian edition, Solution Walkthrough Videos provide easy-to-follow, step-by-step demonstrations of how to solve homework problems.

A new filtering capability in the assignment area allows instructors to customize assignments by using different filters including criteria related to technical applications, level of difficulty, and even learning objectives.

The screenshot shows the 'Create New Assignment' interface in WileyPLUS. It features a search bar at the top, followed by a section titled 'What chapters will this Assignment cover?' with a dropdown menu. Below this are 'Optional Filters' including 'Any Applications', 'Any Study Objective', 'Any Question Type', 'Any Difficulty level', and 'Any Source'. On the right side, there is a 'Mathematical Application' section with a 'Select all' button and a list of application categories with checkboxes, such as 'Numeracy, Measurement (1)', 'Financial (1)', 'Dynamics (1)', 'Electrical, Materials (1)', 'Strength of Materials (1)', 'Electrical (2)', 'Optics and Sound (1)', and 'Thermal (1)'. A 'Find' button is located at the bottom left.

**TECHNICAL MATHEMATICS  
WITH CALCULUS**





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# About the Authors

## THIRD CANADIAN EDITION

**Paul Wraight** is a professor of mathematics and electronics at Durham College in Oshawa, Ontario and has been teaching at the post-secondary level for over 15 years. He is a past president of the Ontario Colleges Mathematics Association and past chair of the Ontario Colleges Mathematics Council. Paul has also provided academic consultation in the development of several mathematics and technical publications. He is a passionate, down-to-earth educator who has been directly involved with several research projects dealing with retention and the at-risk student.

**Sarah White** has five years of teaching experience at Durham College in Oshawa, Ontario in the areas of science and engineering technology and general education. She also provides support and facilitation for the Centre for Academic and Faculty Enrichment at Durham College, in its annual program review and curriculum mapping process. Although she holds an Honours Bachelor of Science degree from the University of Waterloo, math doesn't come easy to Sarah. Reflecting on her own challenges with math has brought tremendous value to this edition as she uses her experiences both in and out of the classroom to focus on presenting content in a straightforward and easy to understand manner. Sarah is an enthusiastic and dedicated educator with a keen interest in educational research and pedagogical approaches to improve and enhance the student learning experience.

## U.S. EDITION

**Paul Calter** is Professor Emeritus of Mathematics at Vermont Technical College and Visiting Scholar at Dartmouth College. A graduate of The Cooper Union, New York, he received his MS from Columbia University and an MFA from Norwich University. Professor Calter has taught technical mathematics for over 25 years. In 1987, he was the recipient of the Vermont State College Faculty Fellow Award.

Professor Calter is a member of the American Mathematical Association of Two Year Colleges, the Mathematical Association of America, the National Council of Teachers of Mathematics, the International Society for the Arts, Sciences, and Technology, the College Art Association, and the Author's Guild.

Professor Calter is the author of several other mathematics textbooks, among which are *Schaum's Outline of Technical Mathematics*, *Practical Math Handbook for the Building Trades*, *Practical Math for Electricity and Electronics*, *Mathematics for Computer Technology*, *Introductory Algebra and Trigonometry*, and *Technical Calculus*.

**Michael Calter** is Associate Professor at Wesleyan University. He received his BS from the University of Vermont. After receiving his Ph.D. from Harvard University, he completed a post-doctoral fellowship at the University of California at Irvine.

Michael has been working on his father's mathematics texts since 1983, when he completed a set of programs to accompany *Technical Mathematics with Calculus*. Since that time, he has become progressively more involved with his father's writing endeavours, culminating with becoming co-author on the second edition of *Technical Calculus* and the fourth edition of *Technical Mathematics with Calculus*.

# Preface

The third Canadian edition of *Technical Mathematics with Calculus* has been significantly revised to incorporate features based on the most current and relevant pedagogical research in mathematics instruction for student success.

We continue to provide instructors with the tools they need to engage learning and teach content in a way that meets the needs of students. We present concepts in a straightforward, accessible manner meant to reduce common math anxiety by focusing on the essentials and providing a painless and easy to grasp learning experience. The text equips students with the necessary tools and problem-solving skills to be successful both in this course and in their chosen fields. The third Canadian edition of this well-respected text has been carefully developed to build upon the success of the second Canadian edition, while introducing a new, fresh approach as suggested by leading educational researchers, reviewers, colleagues, and students.

## FEATURES OF THE CANADIAN EDITION:

- This text is presented **predominantly in SI units**, to complement the national and global use of these units. Where appropriate, there is a small retention of content based on the use of British Imperial units to allow students the opportunity to solve problems in either unitary system of designation.
- **Canadian real-world applications** appear throughout the text to illustrate the relevance and usefulness of technical mathematics outside of the classroom.
- The text reflects Canadian spelling conventions and terminology, and incorporates a Canadian geographical and cultural point of view.

## NEW TO THIS EDITION:

Several areas of *Technical Mathematics with Calculus*, Third Canadian Edition have been thoroughly revised and new features have been added aimed at providing direct instruction to improve student understanding and success. These features break down essential mathematical components for reduced cognitive load and easier understanding. They also motivate students by linking the concepts presented in the pages of the book with their application in the world outside the classroom.

- **Fully revised Chapter 1 with major revisions to chapters 2–10:** The length of Chapter 1 has been significantly reduced from the second edition. Chapters 1–10 have been revised with user-friendly, manageable, compact content and helpful summaries for quick reference. We have focused on direct, step-by-step instruction to address issues commonly experienced by today's students such as blind assumption, math anxiety, and numeracy challenges.
- **Revised Chapter 13:** Sections on exponents and radicals in this chapter have been revised to respond to feedback from instructors and students who have used previous editions of the text.
- **Revised Chapters 27 and 30:** Sections covering derivatives of algebraic functions and integration have been heavily revised to improve and accelerate student understanding and success in these foundational areas of calculus.
- **Reduced anecdotal peripheral information** will lighten the cognitive load and lower the level of distraction for today's student. This aligns with current research-based evidence for teaching and learning.

- **Revised chapter introductions** provide context for students by linking to and building upon material previously discussed in the textbook as well as answering the common questions “Why do I need to know this material and how is it important?”
- **Section and chapter review questions** have generally been reorganized and grouped according to the order in which the material is presented in the text. This will help students refer to the appropriate chapter section if they are experiencing difficulty with a review question.
- **Additional tables** summarize key concepts, definitions, and rules to reduce reading fatigue and page flipping. The tables also provide excellent quick reference tools for review and study.
- **Complete walkouts of example solutions and step-by-step instruction** showcase organizational problem-solving processes and focus student attention on key elements. This feature aligns with current STEM research, emphasizing direct instruction in order to improve math skills.



- **Calculator Tip boxes and new calculator button graphics** show students how calculations might look on their own calculators, helping with visualization and reducing calculator anxiety and frustration.



- **Math Rule icon** appears in the margin next to mathematical rules that are important for students to remember.



- **Math Show Off boxes** provide students with opportunities to further build confidence in content knowledge and show what they know to their peers.



- Revised **Common Error boxes** emphasize some common mistakes or areas of difficulty experienced by students.
- **Four-colour text design and new visual graphics** help to illustrate mathematical relationships and concepts, enhancing visual presentation of material rather than relying entirely on written descriptions.

### PEDAGOGICAL FEATURES:

In addition to the new features added for the third Canadian edition, the authors have built upon a foundation of pedagogical features from previous editions of the text.

- **Learning objectives** are listed at the beginning of each chapter and directly link to each section of the chapter, stating specifically what the student should be able to achieve upon completing
- The numerous **examples** form the backbone of the textbook. They are fully worked out and provide step-by-step instruction to help the student complete the exercises.
- The examples, and often the text itself, include discussions of many **technical applications**. They are included for classes that wish to cover these topics, and also enhance student engagement by showing that mathematics has real uses. The **Index of Applications**, which is printed on the endpapers, will help instructors and students locate specific applications.
- **Problems with approximate solutions** are also available in the text. These include not only expressions and equations with approximate constants, but also those that do not yield to many of the exact methods that are taught, and must be tackled with an approximate method.
- Also included are suggestions on how to **estimate an answer** in order for students to check their work. Suggestions for estimation are provided in the Chapter 3 material on word problems. Following that, many applications examples begin with an estimation step or end with a check, or simply an examination of the answer for reasonableness.

### PROBLEM MATERIAL FEATURES:

- Thousands of exercises are included to give students the essential practice they need to learn mathematics. Exercises are given after each section, graded by difficulty and grouped by type, to allow practice in a particular area.
- **Chapter review problems** cover concepts from the chapter and are generally grouped according to the section in which they appear in the text.
- Answers to all odd-numbered problems are given in **Answers to Selected Problems** at the back of the book.
- Complete solutions to every problem are contained in the **Instructor Solutions Manual**.

**INSTRUCTOR AND STUDENT RESOURCES:**

- **Instructor Solutions Manual** containing worked out solutions to every problem in the text.
- **Test bank** offered in both Word and computerized formats
- **PowerPoint slides** designed to support classroom lectures
- **All appendices** are now available as online resources ([www.wiley.com/go/caltermcanada](http://www.wiley.com/go/caltermcanada)):
  - A. Summary of Facts and Formulas
  - B. Conversion Factors
  - C. Table of Integrals
  - D. Graphing Calculator and Computer Applications

**WileyPLUS**

*WileyPLUS*: An innovative, research-based online environment for effective teaching and learning.

*WileyPLUS* builds students' confidence because it takes the guesswork out of studying by providing students with a clear roadmap: what to do, how to do it, if they did it right. Students will take more initiative so you'll have greater impact on their achievement in the classroom and beyond.

This edition includes **ORION**, an adaptive, personalized learning experience that delivers easy-to-use analytics that allow instructors to see exactly where their students excel and where they need help. ORION helps students to learn by learning about them, and providing practice where they need it most. As a result, students will use their study time more effectively.

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A textbook like this requires an entire team to see it through to completion.

I would like to thank the awesome people at Wiley again for this tremendous opportunity. To Susie Ponte for coming up with the original idea to have me write this book and supporting me through this edition. To Zoë Craig for her support and encouragement for this edition. Finally, to Andrea Grzybowski, taskmaster extraordinaire, for all your gentle reminders that helped to keep us on task.

Thanks to all my colleagues at Durham College, especially Lauren Fuentes and Tony Van Schyndel, for your continuing inspiration and support in this endeavour. To my writing partner and friend Sarah White, for your amazing support and offering the best feedback to me. We make an awesome team!

Again to my parents for all of your gifts including the logic, patience, and creative vision to see beauty in numbers and the ability to problem solve. To my wife Irene and daughter Alyssa (and Patrick) – thank you for all your support and understanding. I love you.

I dedicate this book to all college students to whom math doesn't come easy, to Sarah's son Andrew White, who has taught me so much during this edition, and to the memories of my dad, Doug, and my daughter Brianne.

Paul Wraight, Oshawa, ON

First and foremost, a very special thank you to my colleague and forever friend Paul Wraight for seeing the talent within me and having the confidence in my abilities to make our vision of this textbook a reality. Paul, you are an extraordinary educator and person and I don't know what I would ever do without you in my life. To the team at Wiley, for your faith in Paul to bring me onboard and for providing me with this incredible professional opportunity. To Zoë Craig and Andrea Grzybowski for keeping us on task, on track, and supporting our pedagogical approach. To my parents, Jean and Richard, and my husband Jeff for your ongoing support and love throughout this project and many other journeys. To all of my students, especially my children Madelaine and Andrew: you always inspire me to see the world (including math) through your eyes. This book is dedicated to you.

Sarah White, Oshawa, ON

# Foreword

As Associate Director of the College Mathematics Project (CMP) and the College Student Achievement Project (CSAP), I have had the pleasure of working with Paul Wraight over the past seven years. In his role as President of the Ontario Colleges Mathematics Association (OCMA), Paul has contributed significantly to the success of both the CMP and the CSAP, projects that have involved all 24 of Ontario's Colleges of Applied Arts & Technology as well as its 72 school boards. The research for these projects has included analysis of the mathematics achievement of students in their first year of college and deliberations over ways to improve student success.

The pedagogical approach of *Technical Mathematics with Calculus*, Third Canadian Edition is fully aligned with the recommendations of the CMP and the CSAP, specifically those highlighting the need to increase basic numeracy skills of college students. The author team has been careful to incorporate techniques and features throughout this text to enhance student learning and retention. They accomplish this by focusing the student's attention on key elements and presenting the content in a visually appealing, user friendly, manageable, and compact manner. They have integrated the latest research on STEM teaching and learning with an emphasis on direct instruction to improve math skills.

During our most recent CSAP forum, former college president and deputy minister Dr. Charles Pascal pointed out that, "the best teachers are not 'brainiacs;' they are people who love their subject matter, who had to break it down for themselves, and who want to build a bridge for those whose learning success is their passion." This author team shares this passion for learning success and their text builds bridges for students in a warm and inviting manner. Overall, this book shares our common goal of helping today's students overcome their numeracy skills challenges and advances our common vision of increasing success for all students.

Graham Orpwood, PhD

Professor Emeritus, York University



# Contents

<b>1</b>	<b>Numerical Computation</b>	<b>1</b>
1-1	The Number Types	1
1-2	Numerical Operations	7
1-3	Order of Operations	17
1-4	Scientific and Engineering Notation	20
1-5	Units of Measurement	23
1-6	Substituting into Equations and Formulas	29
1-7	Percentage	31
	Chapter 1 Review Problems	38
<b>2</b>	<b>Introduction to Algebra</b>	<b>40</b>
2-1	Algebraic Expressions	40
2-2	Addition and Subtraction of Algebraic Expressions	45
2-3	Integral Exponents	49
2-4	Multiplication of Algebraic Expressions	56
2-5	Division of Algebraic Expressions	62
	Chapter 2 Review Problems	67
<b>3</b>	<b>Simple Equations and Word Problems</b>	<b>70</b>
3-1	Solving First-Degree Equations	70
3-2	Solving Word Problems	77
3-3	Financial Problems	83
3-4	Mixture Problems	84
3-5	Statics Problems	88
	Chapter 3 Review Problems	91
<b>4</b>	<b>Functions</b>	<b>93</b>
4-1	Functions and Relations	94
4-2	Functional Notation	101
4-3	Composite Functions and Inverse Functions	107
	Chapter 4 Review Problems	111
<b>5</b>	<b>Graphs</b>	<b>112</b>
5-1	Plotting Rectangular Coordinates	112
5-2	Graphing a Function Using Ordered Pairs	115
5-3	Graphing a Linear Function	117
5-4	Graphing Empirical Data, Formulas, and Parametric Equations	120
5-5	Solving Equations with Graphs	122
	Chapter 5 Review Problems	124

---

<b>6</b>	<b>Geometry</b>	<b>125</b>
6-1	Straight Lines and Angles	125
6-2	Polygons	130
6-3	Triangles	131
6-4	Quadrilaterals	137
6-5	Circles	139
6-6	Volumes and Areas of Solids	143
	Chapter 6 Review Problems	146
<hr/>		
<b>7</b>	<b>Right Triangles and Vectors</b>	<b>147</b>
7-1	Angles and Their Measures	147
7-2	The Trigonometric Functions	151
7-3	Finding the Angle When the Trigonometric Function Is Given	155
7-4	Solution of Right Triangles	156
7-5	Applications of the Right Triangle	160
7-6	Vectors	164
7-7	Applications of Vectors	168
	Chapter 7 Review Problems	171
<hr/>		
<b>8</b>	<b>Factors and Factoring</b>	<b>173</b>
8-1	Common Factors	173
8-2	Difference of Two Squares	175
8-3	Factoring Quadratic Trinomials	178
8-4	Factoring by Grouping	182
8-5	The General Quadratic Trinomial	182
8-6	The Perfect Square Trinomial	185
8-7	Sum or Difference of Two Cubes	187
	Chapter 8 Review Problems	188
<hr/>		
<b>9</b>	<b>Fractions and Fractional Equations</b>	<b>190</b>
9-1	Simplification of Fractions	190
9-2	Multiplication and Division of Fractions	195
9-3	Addition and Subtraction of Fractions	197
9-4	Complex Fractions	201
9-5	Fractional Equations	203
9-6	Word Problems Leading to Fractional Equations	205
9-7	Literal Equations and Formula Manipulation	210
	Chapter 9 Review Problems	214
<hr/>		
<b>10</b>	<b>Systems of Linear Equations</b>	<b>216</b>
10-1	Systems of Two Linear Equations	216
10-2	Other Systems of Equations	223
10-3	Word Problems with Two Unknowns	227
10-4	Systems of Three Linear Equations	234
	Chapter 10 Review Problems	239
<hr/>		
<b>11</b>	<b>Determinants</b>	<b>241</b>
11-1	Second-Order Determinants	241
11-2	Third-Order Determinants	246
11-3	Higher-Order Determinants	253
	Chapter 11 Review Problems	258

<b>12</b>	<b>Matrices</b>	<b>260</b>
12-1	Definitions	260
12-2	Operations with Matrices	265
12-3	The Inverse of a Matrix	276
12-4	Solving a System of Linear Equations by Matrix Inversion	279
	Chapter 12 Review Problems	282
<b>13</b>	<b>Exponents and Radicals</b>	<b>284</b>
13-1	Integral Exponents	284
13-2	Simplification of Radicals	288
13-3	Operations with Radicals	293
13-4	Radical Equations	298
	Chapter 13 Review Problems	301
<b>14</b>	<b>Quadratic Equations</b>	<b>302</b>
14-1	Solving Quadratics by Factoring	302
14-2	Solving Quadratics by Completing the Square	308
14-3	Solving Quadratics by Formula	309
14-4	Applications and Word Problems	312
14-5	Graphing the Quadratic Function	314
14-6	Equations of Quadratic Type	316
14-7	Simple Equations of Higher Degree	318
14-8	Systems of Quadratic Equations	319
	Chapter 14 Review Problems	322
<b>15</b>	<b>Oblique Triangles and Vectors</b>	<b>323</b>
15-1	Trigonometric Functions of Any Angle	323
15-2	Law of Sines	331
15-3	Law of Cosines	335
15-4	Applications	339
15-5	Addition of Vectors	342
	Chapter 15 Review Problems	346
<b>16</b>	<b>Radian Measure, Arc Length, and Circular Motion</b>	<b>347</b>
16-1	Radian Measure	347
16-2	Arc Length	354
16-3	Uniform Circular Motion	357
	Chapter 16 Review Problems	359
<b>17</b>	<b>Graphs of the Trigonometric Functions</b>	<b>360</b>
17-1	The Sine Curve	360
17-2	The General Sine Wave, $y = a \sin(bx + c)$	364
17-3	Quick Sketching of the Sine Curve	370
17-4	Graphs of More Trigonometric Functions	372
17-5	The Sine Wave as a Function of Time	377
17-6	Polar Coordinates	384
17-7	Graphing Parametric Equations	388
	Chapter 17 Review Problems	390

<b>18</b>	<b>Trigonometric Identities and Equations</b>	<b>392</b>
18-1	Fundamental Identities	392
18-2	Sum or Difference of Two Angles	397
18-3	Functions of Double Angles	402
18-4	Functions of Half-Angles	404
18-5	Trigonometric Equations	407
18-6	Inverse Trigonometric Functions	413
	Chapter 18 Review Problems	415
<b>19</b>	<b>Ratio, Proportion, and Variation</b>	<b>417</b>
19-1	Ratio and Proportion	417
19-2	Direct Variation	421
19-3	The Power Function	424
19-4	Inverse Variation	431
19-5	Functions of More Than One Variable	434
	Chapter 19 Review Problems	437
<b>20</b>	<b>Exponential and Logarithmic Functions</b>	<b>440</b>
20-1	The Exponential Function	440
20-2	Exponential Growth and Decay	444
20-3	Logarithms	452
20-4	Properties of Logarithms	456
20-5	Exponential Equations	462
20-6	Logarithmic Equations	468
20-7	Graphs on Logarithmic and Semilogarithmic Paper	471
	Chapter 20 Review Problems	477
<b>21</b>	<b>Complex Numbers</b>	<b>479</b>
21-1	Complex Numbers in Rectangular Form	480
21-2	Graphing Complex Numbers	485
21-3	Complex Numbers in Trigonometric and Polar Forms	485
21-4	Complex Numbers in Exponential Form	490
21-5	Vector Operations Using Complex Numbers	493
21-6	Alternating Current Applications	495
	Chapter 21 Review Problems	500
<b>22</b>	<b>Analytic Geometry</b>	<b>501</b>
22-1	The Straight Line	502
22-2	The Circle	513
22-3	The Parabola	519
22-4	The Ellipse	526
22-5	The Hyperbola	534
	Chapter 22 Review Problems	542
<b>23</b>	<b>Binary, Hexadecimal, Octal, and BCD Numbers</b>	<b>544</b>
23-1	The Binary Number System	544
23-2	The Hexadecimal Number System	550
23-3	The Octal Number System	553
23-4	BCD Codes	554
	Chapter 23 Review Problems	555

<b>24</b>	<b>Inequalities and Linear Programming</b>	<b>557</b>
24-1	Definitions	557
24-2	Solving Inequalities	561
24-3	Linear Programming	568
	Chapter 24 Review Problems	572
<b>25</b>	<b>Sequences, Series, and the Binomial Theorem</b>	<b>573</b>
25-1	Sequences and Series	573
25-2	Arithmetic Progressions	578
25-3	Geometric Progressions	584
25-4	Infinite Geometric Progressions	587
25-5	The Binomial Theorem	591
	Chapter 25 Review Problems	599
<b>26</b>	<b>Introduction to Statistics and Probability</b>	<b>601</b>
26-1	Definitions and Terminology	601
26-2	Frequency Distributions	605
26-3	Numerical Description of Data	610
26-4	Introduction to Probability	619
26-5	The Normal Curve	627
26-6	Standard Errors	631
26-7	Process Control	636
26-8	Regression	644
	Chapter 26 Review Problems	648
<b>27</b>	<b>Derivatives of Algebraic Functions</b>	<b>651</b>
27-1	Limits	651
27-2	The Derivative	658
27-3	Rules for Derivatives	666
27-4	Derivative of a Function Raised to a Power	669
27-5	Derivatives of Products and Quotients	672
27-6	Derivatives of Implicit Relations	676
27-7	Higher-Order Derivatives	680
	Chapter 27 Review Problems	680
<b>28</b>	<b>Graphical Applications of the Derivative</b>	<b>682</b>
28-1	Tangents and Normals	682
28-2	Maximum, Minimum, and Inflection Points	685
28-3	Sketching, Verifying, and Interpreting Graphs	693
28-4	Approximate Solution of Equations by Newton's Method	696
	Chapter 28 Review Problems	698
<b>29</b>	<b>Applied Applications of the Derivative</b>	<b>699</b>
29-1	Rate of Change	699
29-2	Motion of a Point	703
29-3	Related Rates	709
29-4	Optimization	714
	Chapter 29 Review Problems	722

<b>30</b>	<b>Integration</b>	<b>724</b>
30-1	The Indefinite Integral	724
30-2	Rules for Finding Integrals	731
30-3	Constant of Integration	736
30-4	The Definite Integral	738
30-5	Approximate Area Under a Curve	740
30-6	Exact Area Under a Curve	745
	Chapter 30 Review Problems	747
<b>31</b>	<b>Applications of the Integral</b>	<b>748</b>
31-1	Applications to Motion	748
31-2	Applications to Electric Circuits	752
31-3	Finding Areas by Means of the Definite Integral	754
31-4	Volumes by Integration	762
	Chapter 31 Review Problems	767
<b>32</b>	<b>More Applications of the Integral</b>	<b>769</b>
32-1	Length of Arc	769
32-2	Area of Surface of Revolution	772
32-3	Centroids	774
32-4	Fluid Pressure	781
32-5	Work	783
32-6	Moment of Inertia	786
	Chapter 32 Review Problems	791
<b>33</b>	<b>Derivatives of Trigonometric, Logarithmic, and Exponential Functions</b>	<b>792</b>
33-1	Derivatives of the Sine and Cosine Functions	792
33-2	Derivatives of the Tangent, Cotangent, Secant, and Cosecant Functions	798
33-3	Derivatives of the Inverse Trigonometric Functions	802
33-4	Derivatives of Logarithmic Functions	804
33-5	Derivatives of the Exponential Function	809
	Chapter 33 Review Problems	813
<b>34</b>	<b>Methods of Integration</b>	<b>814</b>
34-1	Integrals of Exponential and Logarithmic Functions	814
34-2	Integrals of the Trigonometric Functions	817
34-3	Average and Root Mean Square Values	819
34-4	Integration by Parts	821
34-5	Integrating Rational Fractions	824
34-6	Integrating by Algebraic Substitution	830
34-7	Integrating by Trigonometric Substitution	833
34-8	Improper Integrals	836
34-9	Approximate Value of a Definite Integral	838
	Chapter 34 Review Problems	844
<b>35</b>	<b>Differential Equations</b>	<b>845</b>
35-1	Definitions	845
35-2	Graphical and Numerical Solution of Differential Equations	848
35-3	First-Order Differential Equation, Variables Separable	851
35-4	Exact First-Order Differential Equations	854



- 35-5 First-Order Homogeneous Differential Equations 856
- 35-6 First-Order Linear Differential Equations 858
- 35-7 Geometric Applications of First-Order Differential Equations 863
- 35-8 Exponential Growth and Decay 866
- 35-9 Series *RL* and *RC* Circuits 868
- 35-10 Second-Order Differential Equations 873
- 35-11 Second-Order Differential Equations with Constant  
Coefficients and Right Side Zero 874
- 35-12 Second-Order Differential Equations with Right Side Not Zero 880
- 35-13 *RLC* Circuits 885
- Chapter 35 Review Problems 892

---

## **36 Solving Differential Equations by the Laplace Transform and by Numerical Methods 894**

- 36-1 The Laplace Transform of a Function 894
- 36-2 Inverse Transforms 901
- 36-3 Solving Differential Equations by the Laplace Transform 903
- 36-4 Electrical Applications 907
- 36-5 Numerical Solution of First-Order Differential Equations 911
- 36-6 Numerical Solution of Second-Order Differential Equations 914
- Chapter 36 Review Problems 917

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## **37 Infinite Series 919**

- 37-1 Convergence and Divergence of Infinite Series 920
- 37-2 Maclaurin Series 922
- 37-3 Taylor Series 929
- 37-4 Operations with Power Series 933
- 37-5 Fourier Series 936
- 37-6 Waveform Symmetries 942
- 37-7 Waveforms with Period of  $2L$  946
- 37-8 A Numerical Method for Finding Fourier Series 949
- Chapter 37 Review Problems 951

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## **Answers to Selected Problems 953**

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## **Index i-1**





TABLE 1-1 Number Types (continued)

Number Type	Definition	Example	Explanation
Rational Numbers	Includes integers and all other numbers that can be expressed as the <i>quotient</i> of two integers.	$\frac{1}{2}, -\frac{3}{5}, \frac{57}{23}, -\frac{98}{99}, 7$	A <i>quotient</i> is a number formed by dividing one number by another number.
Irrational Numbers	Numbers that <i>cannot</i> be expressed as the quotient of two integers.	$\sqrt{2}, \sqrt[3]{5}, \sqrt{7}, \pi$	These are numbers that, written as decimals, are nonterminating and nonrepeating.
Real Numbers	Rational and irrational numbers combined together into one set of numbers.	Any number other than a complex number	Rational and irrational numbers are two separate number types until we put them together into a single set of “real” numbers.
Complex Numbers	Any <i>imaginary</i> number (non-real) or a combination of a real number and an imaginary number	Any imaginary number like $\sqrt{-4}$ , and any combination like $-4 + \sqrt{-9}$	When we square a real number, even a negative one, we get a positive number. So, taking the square root of a negative number doesn’t make sense as a real number, and the result is an <i>imaginary number</i> .

Note that all of the numbers we will work with in Chapter 1 are real numbers. (Complex numbers are covered in Chapter 21.)

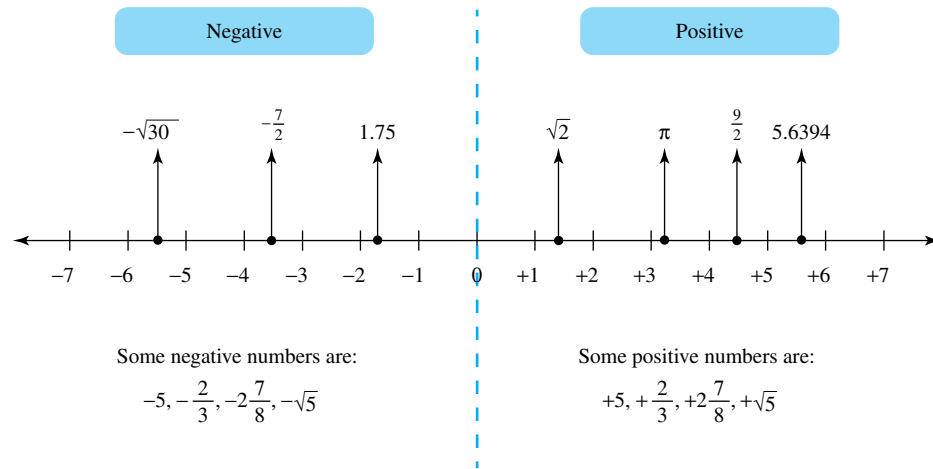


FIGURE 1-1

### Positional Number Systems

A *positional number system* is one which the position of a digit determines its value. Our decimal system is positional. Table 1-2 shows the values of positions in a decimal number.

### Place Value

Each position in a number has a *place value* equal to the base of the number system raised to the power of the position number. The place values in the decimal number system, as well as the place names, are shown in Table 1-2.

TABLE 1-2 Decimal Position Values

Number	Power of 10	Name
10 000	$10^4$	Ten Thousands
1 000	$10^3$	Thousands
100	$10^2$	Hundreds
10	$10^1$	Tens
1	$10^0$	Ones
0	0	Zero
0.1	$10^{-1}$	Tenths
0.01	$10^{-2}$	Hundredths
0.001	$10^{-3}$	Thousandths
0.0001	$10^{-4}$	Ten Thousandths

### The Opposite of a Number

The *opposite* of a number  $n$  is the number which, when added to  $n$ , gives a sum of zero.

◆◆◆ **Example 1:** The opposite of 2 is  $-2$ , because  $2 + (-2) = 0$ . The opposite of  $-6$  is  $+6$ . ◆◆◆

Geometrically, the opposite  $-n$  of a number  $n$  lies on the opposite side of the zero point of the number line from  $n$ , and at an equal distance from the zero point (see Fig. 1-1). The opposite of a number is also called the *additive inverse* of that number.

### Symbols of Equality and Inequality

Several symbols are used to show the relative positions of two quantities  $a$  and  $b$  on the number line.

$a = b$  means that  $a$  equals  $b$  and that  $a$  and  $b$  occupy the same position on the number line.

$a \neq b$  means that  $a$  and  $b$  are *not equal* and have different locations on the number line.

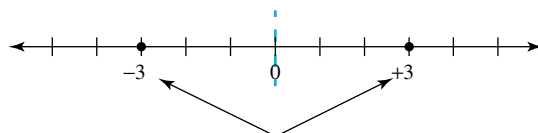
$a > b$  means that  $a$  is greater than  $b$  and  $a$  lies to the right of  $b$  on the number line.

$a < b$  means that  $a$  is less than  $b$  and  $a$  lies to the left of  $b$  on the number line.

$a \approx b$  means that  $a$  is approximately equal to  $b$  and that  $a$  and  $b$  are near each other on the number line.

### Absolute Value

To get a good understanding of what *absolute value* means, let's look at a number line:



If both of these integers are inside “absolute value” bars, then the result is the same: either  $|-3|$  or  $|+3| = 3$ .

FIGURE 1-2

The *absolute value* or *magnitude* of a number  $n$  is defined as its distance from zero on a number line, regardless of its algebraic sign. Therefore, absolute value is *never* negative. The absolute value of  $n$  is written between bars:  $|n|$ .

**COMMON ERROR**

Absolute value bars  $| |$  around signed numbers are often confused with brackets. They do *not* work like parentheses, brackets, or braces with negative signs.

**Remember:**  $-|-4| = -4$ , *not*  $+4$ , as you would normally simplify  $-(-4)$ .

◆◆◆ **Example 2:** Find the value of  $-|3 - 8|$ .

Solution:

Instruction		Looks Like
Step 1	Simplify the expression inside the absolute value bars: $3 - 8 = -5$ .	$- -5 $
Step 2	Convert the absolute value bars around $ -5 $ to a positive sign, and replace the absolute value bars with parentheses.	$-(+5)$
Step 3	Remove the parentheses and associate the negative sign to the number.	$-5$

We can see that the opposite of an absolute value is a negative number (or zero). ◆◆◆

◆◆◆ **Example 3:** Find each absolute value or combination of absolute values.

- $|5| = 5$
- $|-9| = (+9)$  or 9
- $|3 - 7| = |-4| = (+4)$  or 4
- $-|-4 + 7| = -|+3| = -(+3) = -3$
- $-|7 - 21| - |19 - 13| = -|-14| - |+6| = -(+14) - (+6) = -14 - 6 = -20$  ◆◆◆

**Approximate Numbers**

Most of the numbers we deal with in the fields of science and technology are *approximate*, meaning that their value is somewhat uncertain.

◆◆◆ **Example 4:**

- All numbers that represent *measured* quantities are approximate. A certain shaft, for example, is approximately 1.75 cm in diameter.
- Many *fractions* can be expressed only approximately in decimal form. Thus,  $\frac{2}{3}$  is approximately equal to 0.6667.
- Irrational numbers* can be written only approximately in decimal form. The number  $\sqrt{3}$  is approximately equal to 1.732. ◆◆◆

**Exact Numbers**

*Exact* numbers are those that have no uncertainty.

◆◆◆ **Example 5:**

- There are exactly 24 hours in a day; no more, no less.
- An automobile has exactly four wheels.
- Exact numbers are usually integers, but not always. For example, an inch is *exactly* 25.4 mm, by definition.
- On the other hand, not all integers represent exact amounts. For example, a certain town has a population of *approximately* 3500 people. ◆◆◆



### Significant Digits

Zeros are frequently used in decimal numbers as “placeholder zeros” in order to determine the location of the decimal point. These zeros are *not significant*. The remaining numbers (including any zeros in between them) are called *significant digits* (or sometimes significant figures).

◆◆◆ **Example 6:** Table 1-3 outlines some rules to follow when working with significant digits, along with examples of each rule.

**TABLE 1-3** Rules for Significant Digits

Rule	Examples	
All nonzero digits are significant.	497.3 37.8 18	has 4 significant digits has 3 significant digits has 2 significant digits
Zero digits to the left of the first nonzero digit are not significant and are known as “placeholder zeros” to locate the decimal point.	0.0003 0.0583 0.000 583	has 1 significant digit. has 3 significant digits. has 3 significant digits.
Zero digits located between nonzero digits are significant. This is sometimes called the “bookend case” because the zero is “bookended” by nonzero digits.	39.05 803 2.0008	has 4 significant digits. has 3 significant digits. has 5 significant digits.
Zeros to the right of the decimal point after a number are significant. They are not needed to locate the decimal point, but show that those digits are zeros because they have been measured and are important.	8.50 1.4900 2.0	has 3 significant digits. has 5 significant digits. has 2 significant digits.
Zeros at the end of a number with an “implied decimal” are usually not significant. If the decimal point is present, these zeros are significant. An overbar is also sometimes used to show the last significant zero.	10 000 10 000. 10 $\overline{000}$	has 1 significant digit. has 5 significant digits. has 3 significant digits.

◆◆◆

### Accuracy and Precision

The *accuracy* of a number is written as the number of significant digits in the number. The *precision* of a number is written as the position of the rightmost significant digit.

◆◆◆ **Example 7:**

Number	Accurate to:	Precise to:
1.255	4 significant digits	thousandths
23 800	3 significant digits	hundreds
0.002	1 significant digit	thousandths
3.600	4 significant digits	thousandths

◆◆◆

### Rounding

When asked to round your answer after a computation, it is important to follow some basic rules. *Rounding down* means that when the first discarded digit is less than 5, the last retained digit does not change. *Rounding up* means that when the first discarded digit is 5 or more, the last retained digit is increased by 1. There are different rules for rounding when dealing with statistics or accounting, which will not apply in this textbook.

## ◆◆◆ Example 8:

Number	Rounded to Two Decimal Places	Rounded to Two Significant Digits
8.3654...	8.36   <del>5</del> 4 ... 8.37 (rounded up)	8.3   <del>6</del> 54 ... 8.4 (rounded up)
8.3456...	8.35	8.3
7.365 01...	7.37	7.4
14.364 999...	14.36	14
142.764 999...	142.76	140
3.141 592...	3.14	3.1

Note that “...” to the right of a decimal number shows that more digits follow to the right, which are not shown. ◆◆◆

## ◆◆◆ Example 9:

Number	Rounded to Three Decimal Places	Rounded to Three Significant Digits
4.3654	4.365	4.37
4.3656	4.366	4.37
4.365 501	4.366	4.37
4.365 499	4.365	4.37
1.764 999	1.765	1.76
1.764 499	1.764	1.76
-8.3499	-8.350	-8.35
-8.3599	-8.360	-8.36

◆◆◆

## Exercise 1 ♦ The Number Types

## Equality and Inequality Signs

Insert the proper equality or inequality sign ( $=$ ,  $\approx$ ,  $>$ , or  $<$ ) between each pair of numbers.

- 7 and 10
- 9 and  $-2$
- $-3$  and 4
- $-3$  and  $-5$
- $\frac{3}{4}$  and 0.75
- $\frac{2}{3}$  and 0.667

## Absolute Value

Evaluate the expression.

- $-|9 - 23| - |-7 + 3|$
- $-|7 + 45| - |-8 - 34|$
- $|12 - 5 + 8| - |-6| + |15|$
- $|13 - 6 + 9| - |-8| + |13|$
- $-|3 - 9| - |5 - 11| + |21 + 4|$
- $-|4 - 8| + |-5 + 11| - |-12 - 6|$

## Significant Digits

State the number of significant digits in each approximate number.

- 78.3
- 925.3
- 9274
- 29 471
- 4.008
- 5.0004
- 9400
- 36 000
- 20 000.
- $80\bar{0}$  000
- 5000.0
- 60 000.0
- 0.9972
- 0.875 32
- 1.0000
- 63.0000

Round each number to two decimal places.

- 38.468
- 1.996
- 96.835 001
- 55.8650
- 398.372
- 2.9573
- 2985.339
- 278.382

Round each number to one decimal place.

37. 13.98

41. 398.36

38. 745.62

42. 34.927

39. 5.6501

43. 9839.2857

40. 0.482

44. 0.847

Round each number to three significant digits.

49. 9.284

53. 0.083 75

50. 2857

54. 29.555

51. 0.048 25

55. 29.450 01

52. 483 982

56. 8372

Round each number to the nearest hundred.

45. 28 583

47. 3 845 240

46. 7550

48. 274 837

## 1-2 Numerical Operations

A *numerical operation* can be described as an action or process used to solve a numerical problem. The most basic numerical operations are addition, subtraction, multiplication, and division. Exponents and roots are also numerical operations. Let's review the rules of these operations.

### Addition and Subtraction

TABLE 1-4 Addition and Subtraction

Numerical Operation	Rule	Example	Explanation
Addition	Signs	$a + (-b) = a - b$	When adding a negative number $-b$ to a number $a$ , subtract $b$ from $a$ .
	Commutative Law	$a + b = b + a$	You can add numbers in any order.
	Associative Law	$a + (b + c) = (a + b) + c$	You can group numbers to be added in different ways.
	Approximate Numbers	$32.4 \text{ cm} + 5.825 \text{ cm} = 38.2 \text{ cm}$ (not $\approx 38.225 \text{ cm}$ )	When adding approximate numbers, keep as many decimal places in your answer as the number having the <i>fewest</i> decimal places or significant digits in the question. Do not use the $\approx$ symbol.
Subtraction	Signs	$a - (-b) = a + b$	When subtracting a negative number $-b$ from a number $a$ , you add $b$ to $a$ .
	Approximate Numbers	$79.434 \text{ m} - 8.9954 \text{ m} = 70.439 \text{ m}$ (not $\approx 70.4386 \text{ m}$ )	When subtracting approximate numbers, keep as many decimal places in your answer as the number having the <i>fewest</i> decimal places or significant digits in the question. Do not use the $\approx$ symbol.

### Adding Signed Numbers

#### ◆◆◆ Example 10:

(a)  $7 + (-2) = 7 - 2 = 5$

(b)  $-8 + (-3) = -8 - 3 = -11$

(c)  $9.92 \text{ m} + (-15.36 \text{ m}) = 9.92 \text{ m} - 15.36 \text{ m} = -5.44 \text{ m}$

◆◆◆

### Subtracting Signed Numbers

#### ◆◆◆ Example 11:

- (a)  $15 - (-3) = 15 + 3 = 18$   
 (b)  $-5 - (-9) = -5 + 9 = 4$   
 (c)  $-25.62 - (-5.15) = -25.62 + 5.15 = -20.47$

◆◆◆

### Subtracting Negative Numbers by Calculator

#### COMMON ERROR



Be careful when calculating signed numbers. Calculators often have two separate buttons with the 'negative' sign. The  $-$  button is for the operation of subtraction and the  $(-)$  or  $+/-$  button on most calculators is for changing signs on signed numbers.

### Addition and Subtraction of Approximate Numbers

It is a good practice to keep all the digits given throughout the calculation process, until your final answer is needed. Generally, for multiple calculation problems, your calculator will keep all the digits for you if you use the ANS button.

Don't round given numbers or calculations *before* computing your final calculation. Round the final calculated answer as the problem requires.

#### ◆◆◆ Example 12:

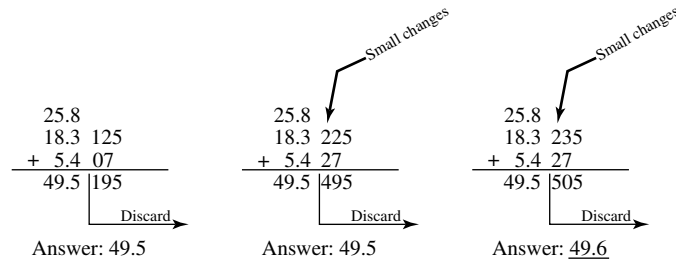


FIGURE 1-3

◆◆◆

◆◆◆ **Example 13:** A sports stadium contains about 3500 people. It starts to rain and 372 people leave. How many are left in the stadium?

**Solution:** Subtracting, we obtain

$$3500 - 372 = 3128$$

We round our answer to 3100 people, because the word “about” tells you that 3500 likely has only two significant digits.

◆◆◆

### Adding and Subtracting Exact and Approximate Numbers

When combining an exact number with an approximate number, the approximate number will limit the accuracy of the result. You will need to round the answer to the number of decimal places found in the approximate number, even if the exact number has fewer decimal places.

◆◆◆ **Example 14:** Express 2 h and 35.8 min in minutes.

**Solution:** We must add an exact number, 120 min, and an approximate number, 35.8 min:

$$120 \text{ min} + 35.8 \text{ min} = 155.8 \text{ min}$$

Since 120 is an exact number, we do not round our answer to the nearest 10-min interval; instead we retain as many decimal places as in the approximate number. So our answer is 155.8 minutes.

◆◆◆

## Multiplication and Division

The numbers we multiply to get a *product* are called *factors*. For example,

$$\begin{array}{c} 3 \times 5 = 15 \\ \uparrow \quad \uparrow \quad \uparrow \\ \text{Factors} \quad \quad \quad \text{Product} \end{array}$$

◆◆◆ **Example 15:** Use a calculator to multiply 183 by 27.

**Solution:** You should get  $183 \times 27 = 4941$ . ◆◆◆

**TABLE 1-5** Multiplication and Division

Numerical Operation	Rule	Example	Explanation
Multiplication	Signs	$(+a)(+b) = (-a)(-b) = +ab$ $(+a)(-b) = (-a)(+b) = -ab$ $(+a)(-b)(+c)(-d) = +abcd$ $(-a)(-b)(-c) = -abc$	<p>If <math>a</math> and <math>b</math> have the <i>same sign</i>, the product is signed <math>+</math>.</p> <p>If <math>a</math> and <math>b</math> have <i>different signs</i>, the product is signed <math>-</math>.</p> <p>When multiplying a string of numbers together, if an <i>even</i> number of factors is negative, the answer will be <i>positive</i>. If an <i>odd</i> number of factors are negative, the answer will be <i>negative</i>.</p>
	Commutative Law	$ab = ba$	The <i>order</i> of multiplication is not important.
	Associative Law	$a(bc) = (ab)c = (ac)b = abc$	We can <i>group</i> the numbers to be multiplied any way we want.
	Distributive Law	$a(b + c) = ab + ac$	You can multiply a factor <i>either</i> by adding the group of numbers in the brackets first <i>or</i> by multiplying each factor separately, then adding the products together.
Approximate Numbers	$123.56 \times 2.21 = 273$ (273.0676) $\uparrow \quad \quad \uparrow \quad \quad \uparrow$ five digits    three digits    keep three digits	When multiplying two or more approximate numbers, round the answer to as many significant digits as the factor having the <i>fewest</i> significant digits in the question. Do not use the $\approx$ symbol.	
Exact Numbers	<p>If a certain car tire weighs 32.2 kg when mounted, how much will four of these tires weigh?</p> <p>Solution: <math>32.2(4) = 128.8</math> kg</p> <p>Since 4 is an <i>exact</i> number, we keep as many significant digits as contained in 32.2 kg and round our answer to 129 kg.</p>	When using <i>exact</i> numbers in a computation, treat them as if they had <i>more</i> significant digits than any of the approximate numbers in that computation.	

(continued)